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DETERMINATION OF SHIP COURSE AND SPEED FROM TRANSMITTED SIGNAL --ETC(U)
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(14) TRACOR-DOCUMENT NO. San Diego
SD-67-0007-C
Contract #N123(953)54996A
TRACOR Project No. 002-009-23 (15)

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UNCLASSIFIED

LEVEL II

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(9) TECHNICAL NOTE

(6) DETERMINATION OF SHIP COURSE AND SPEED FROM TRANSMITTED SIGNAL FREQUENCY. (U)

(10) Richard A. Mueller

Submitted To

Conformal/Planar Array
Project Office
Code 2110 - USNEL

(12) 27p.

(11) 22 March ~~22~~ 1967

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TRACOR Document Number
SD-67-0007-C

TECHNICAL NOTE

DETERMINATION OF SHIP COURSE AND SPEED FROM TRANSMITTED SIGNAL FREQUENCY (U)

Submitted To
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March 22, 1967

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1. INTRODUCTION

In the design of the C/P Array Sonar, some consideration is being given to the use of an own-doppler-nullifier technique in which the transmitted frequency is varied according to the speed of the sonar vessel and the direction of the transmitted beam. The objective here is merely to determine if transmitting a frequency which is a function of own ship's course and speed furnishes a valuable additional clue which might enable the enemy submarine to calculate the surface vessel's course and speed. It is not the purpose of this paper to evaluate the merits of the proposed own-doppler-nullifier technique as they apply to signal processing aboard the surface vessel.

The submarine currently has means for determining the course and speed of another vessel. The Torpedo Data Computer (TDC) in the process of solving for torpedo gyro angle, solves for target course and speed in a trial and error manner as a function of time. However, the TDC requires continuing target bearing information, which takes time, and at least an occasional range reading, which entails risk. It would be very attractive and useful to the submarine if the course and speed of a surface vessel could be determined merely by observing the frequency of a transmitted signal. This would be particularly true at long ranges when the submarine might be jockeying for position.

The analysis presented in this paper at all times assumes that the C/P Array Sonar is transmitting pulses which are at a fixed frequency for the duration of any one pulse. An analysis assuming frequency swept pulses would be much more cumbersome with no reason to expect that it would produce a different result. It is also assumed that both the surface vessel and the submarine remain on a straight-line course at constant speed during any interval of observation on which calculations are based.

It is further assumed that the submarine makes all readings of signal frequency and bearing to the surface vessel when the surface vessel is aiming a narrow beam transmission directly at the submarine. The extent to which side-lobe frequency readings might assist the submarine in calculating surface vessel course and speed will be considered in a subsequent paper.

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2. TRANSMIT AND RECEIVE DOPPLER FREQUENCY SHIFTS

The frequency at which a signal is transmitted in the water will be shifted from the frequency at which the transducer array is driven if the array is in motion with respect to the water. The frequency in the water will be

$$f_w = f_D \times \frac{c}{c - v \cos \theta}$$

where f_w = signal frequency in water

f_D = frequency at which array is driven

c = speed of sound in water

v = own-ship speed

θ = direction of transmitted beam measured from own-ship's bow.

This equation can be rewritten in the form

$$f_w = f_D \left[1 + \frac{v}{c} \cos \theta + \frac{v^2}{c^2} \cos^2 \theta + \text{------} \right] .$$

For all likely ship speeds the second degree and higher terms have negligible value so that the frequency of the signal in the water is very nearly equal to

$$f_w \approx f_D \left[1 + \frac{v}{c} \cos \theta \right] \qquad (1)$$

A received echo will be doppler-shifted in the same way as a transmitted signal. Thus, if the receiving array is in motion, the frequency at the output of the array will be given by

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$$f_R \approx f_w \left[1 + \frac{v}{c} \cos \theta\right] \tag{2}$$

where f_R = frequency at output of receiving array

f_w = frequency of signal in water at input to receiving array

v = own-ship speed

c = speed of sound in water

θ = direction of received signal measured from own-ship's bow.

If the signal has been returned by a target which was not in motion, the f_w in equation (2), the frequency of the signal at the input to the receiving array, will be equal to the f_w in equation (1), the frequency of the signal at the output of the transmitting array. Also, the direction to the target on transmission, θ in equation (1), can be expected to be very nearly equal to the direction to the target on reception, θ in equation (2), since the speed of advance of the ship will be very small compared to the speed of sound in water. Thus, the equation for the frequency of the signal at the output of receiving array can be written

$$f_R \approx f_D \left[1 + \frac{v}{c} \cos \theta\right] \left[1 + \frac{v}{c} \cos \theta\right]$$

or

$$f_R \approx f_D \left[1 + \frac{2v}{c} \cos \theta + \frac{v^2}{c^2} \cos^2 \theta\right]$$

For reasonable ship speeds this is very nearly equal to

$$f_R \approx f_D \left[1 + \frac{2v}{c} \cos \theta\right] \tag{3}$$

where f_R = frequency at output of receiving array

f_D = frequency at which transmitting array is driven

v = own-ship speed

c = speed of sound in water

θ = bearing from own-ship to target

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3. DETERMINATION OF SURFACE VESSEL COURSE AND SPEED WITH FULL DOPPLER NULLIFICATION

If own-ship modifies the frequency f_D at which the transmitting array is driven to

$$f_D = \frac{f_c}{1 + \frac{2v}{c} \cos \theta}$$

then the output of the receiving array will be constant at f_c, an arbitrary value from which all frequency deviations will be due solely to target motion. Thus, by modifying the frequency at which the array is driven according to the speed of the ship and the direction of the beam, the received frequency can be made independent of own ship's motion. Under this condition, the frequency of the signal actually transmitted into the water becomes

$$f_w \approx \frac{f_c}{1 + \frac{2v}{c} \cos \theta} \left[1 + \frac{v}{c} \cos \theta\right]$$

or

$$f_w \approx f_c \left[1 - \frac{v}{c} \cos \theta\right] \tag{4}$$

It should be noted that the driving frequency, f_D, is a function of the direction, θ, of the beam. If a broad beam were used, obviously own-doppler could be nullified for only one direction within the beam. However, own-doppler in other directions within the beam would be reduced.

Equation (4) shows the signal frequency the submarine would hear if the sonar were being operated to give a fixed frequency at the output to the receiving array and the submarine lay motionless in the water. If the submarine were in motion, the frequency of the signal heard by the submarine would be

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$$f_{RS} = f_c \left[1 - \frac{v}{c}\cos\theta\right]\left[1 + \frac{v_s}{c}\cos\varphi\right]$$

or

$$f_{RS} \approx f_c \left[1 - \frac{v}{c}\cos\theta + \frac{v_s}{c}\cos\varphi\right] \tag{5}$$

Solving for v gives

$$v \approx \frac{(f_c - f_{RS})\,c}{f_c \cos\theta} + \frac{v_s \cos\varphi}{\cos\theta} \tag{6}$$

where v = surface ship speed

v_s = submarine speed

f_c = constant frequency to which output of surface ship receiving array is held

f_{RS} = frequency of signal received at output of submarine receiving array

c = speed of sound in water

φ = relative bearing from submarine to surface ship

θ = relative bearing from surface ship to submarine

The relative bearing from the surface ship to the submarine, θ, is related to the surface ship's absolute heading by the following expression

$$\theta = \beta + \varphi - \alpha - \pi \tag{7}$$

where α = absolute heading of surface ship

β = absolute heading of submarine

π = 180^o

Equation (6) is then, an expression for surface ship speed in terms of, among other things, the surface ship's course. If it is assumed that values can be found for the other variables,

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the submarine still has two unknowns and only one equation. Unless some further information can be added to the picture, the submarine can solve for neither surface ship speed or course.

Fortunately, there is one other equation that can be written; the change in bearing from the surface ship to the submarine will always be equal to the change in bearing from the submarine to the surface vessel

$$\theta_2 - \theta_1 = \varphi_2 - \varphi_1$$

or

$$\theta_2 = \theta_1 + \varphi_2 - \varphi_1 = \theta_1 + \Delta\varphi \tag{8}$$

where θ_1 = bearing from ship to sub at time t_1

θ_2 = bearing from ship to sub at time t_2

φ_1 = bearing from sub to ship at time t_1

φ_2 = bearing from sub to ship at time t_2

Solving equation (5) for $\cos\theta$, the following two equations can be written for times t_1 and t_2

$$\cos\theta_1 \approx \frac{1}{v}\left[\left(1 - \frac{f_{RS1}}{f_c}\right) c + v_s \cos\varphi_1\right] \tag{9}$$

$$\cos\theta_2 \approx \frac{1}{v}\left[\left(1 - \frac{f_{RS2}}{f_c}\right) c + v_s \cos\varphi_2\right] \tag{10}$$

where it is assumed that the velocity of the surface vessel remains constant at v and that of the submarine remains constant at v_s. Letting

$$k_1 = \left(1 - \frac{f_{RS1}}{f_c}\right) c + v_s \cos\varphi_1 \tag{11}$$

and $$k_2 = \left(1 - \frac{f_{RS2}}{f_c}\right) c + v_s \cos \varphi_2 \tag{12}$$

the two equations can be written more simply as

$$\cos \theta_1 \approx \frac{k_1}{v} \tag{13}$$

$$\cos \theta_2 \approx \frac{k_2}{v} \tag{14}$$

From (8) it can be written

$$\cos \theta_2 = \cos(\theta_1 + \Delta\varphi)$$

But, $$\cos(\theta_1 + \Delta\varphi) = \cos \theta_1 \cos(\Delta\varphi) - \sin \theta_1 \sin(\Delta\varphi)$$

So, $$\frac{\cos \theta_2}{\cos \theta_1} = \frac{\cos \theta_1 \cos(\Delta\varphi) - \sin \theta_1 \sin(\Delta\varphi)}{\cos \theta_1} \approx \frac{k_{2/v}}{k_{1/v}}$$

$$\frac{k_2}{k_1} \approx \cos(\Delta\varphi) - \sin(\Delta\varphi) \tan \theta_1$$

or $$\tan \theta_1 \approx \frac{\cos(\Delta\varphi) - {}^{k_2}\!/_{k_1}}{\sin(\Delta\varphi)}$$

Thus, $$\theta_1 \approx \tan^{-1} \left\{ \frac{\cos(\Delta\varphi) - {}^{k_2}\!/_{k_1}}{\sin(\Delta\varphi)} \right\} \tag{15}$$

From (7) the heading of the surface ship, α, will then be

$$\alpha \approx \beta + \varphi_1 - \pi - \tan^{-1} \left\{ \frac{\cos(\Delta\varphi) - {}^{k_2}\!/_{k_1}}{\sin(\Delta\varphi)} \right\} \tag{16}$$

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and from (13) the velocity, v, will be

$$v \approx \frac{k_1}{\cos \theta_1} \tag{17}$$

where
$$k_1 = \left(1 - \frac{f_{RS1}}{f_c}\right) c + v_s \cos \varphi_1 \tag{11}$$

and β = absolute heading of submarine

φ_1 = relative bearing from submarine to surface ship at time t_1

φ_2 = relative bearing from submarine to surface ship at time t_2

$\Delta\varphi$ = $\varphi_2 - \varphi_1$

π = π radians or 180 degrees

f_{RS1} = frequency of signal at time t_1 measured at output of submarine receiving array

f_{RS2} = frequency of signal at time t_2 measured at output of submarine receiving array

f_c = frequency to which echo from stationary target is held at output of surface vessel receiving array

c = speed of sound in water

v_s = submarine speed

3.1 EXAMPLE

The following example, illustrated in Fig. 3-1, shows how the derived equations could be used by a submarine to calculate a C/P Array Sonar equipped vessel's course and speed.

3.1.1 Calculation of Frequencies Heard by Submarine

v = 25 knots = 42.22 ft/sec Ship speed

α = 0° Ship course

θ_1 = 30° Relative bearing to sub--first observation

θ_2 = $36^{\circ}\ 9'$ Relative bearing to sub--second observation

f_c = 2500 cps Frequency at output to receiving array

v_s = 10 knots = 16.89 ft/sec Sub speed

β = 240° Sub course

φ_1 = 330° Relative bearing to ship--first observation

φ_2 = $336^{\circ}\ 9'$ Relative bearing to ship--second observation

c = 4900 ft/sec Speed of sound in water

The frequency of the signal heard by the submarine on the first and second observations is calculated from Eq. (5) and is found to be

$$f_{RS1} = 2488.81 \text{ cps}$$

$$f_{RS2} = 2490.50 \text{ cps}$$

3.1.2 Calculation by Submarine of Ship's Course and Speed

The following information will be available to the submarine by direct measurement

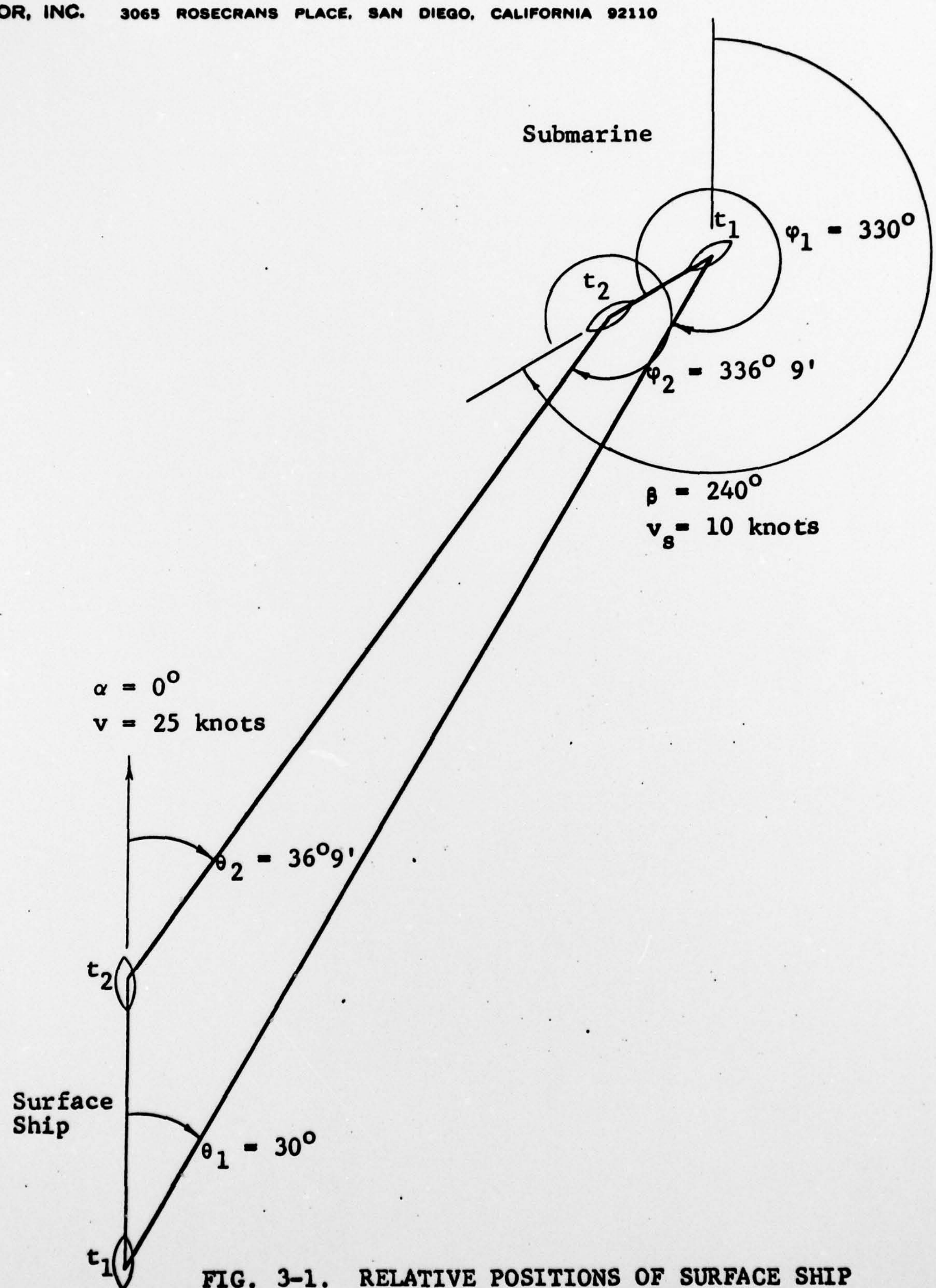

FIG. 3-1. RELATIVE POSITIONS OF SURFACE SHIP AND SUBMARINE AT TIMES t_1 AND t_2

v_s = 10 knots = 16.89 ft/sec. Sub speed

β = 240° Sub course

φ_1 = 330° Relative bearing to ship--first observation

φ_2 = $336^{\circ}\ 9'$ Relative bearing to ship--second observation

c = 4900 ft/sec. Speed of sound in water

f_{RS1} = 2488.81 cps Signal frequency--first observation

f_{RS2} = 2490.50 cps Signal frequency--second observation

It is further assumed that the submarine will know from intelligence sources that the surface vessel is holding the output of its receiver array, f_c, to 2,500 cps.

The above values when introduced into equation (16) yield

$$\alpha = 359^{\circ}\ 45' \quad \text{Surface ship heading}$$

This compares with an actual heading of 0°.

When the input values are introduced into equation (17)

$$v = 24.997 \text{ knots} \quad \text{Surface ship speed}$$

This compares with an actual speed of 25 knots.

Since there is some question as to whether or not the submarine will be able to measure the frequency of the incoming signal to within one-hundreth of a cycle and the signal bearing to within one minute of arc, the above example has been recalculated for measurements accurate to the nearest cycle and nearest degree. Thus,

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$f_{RS1} = 2489$ cps

$f_{RS2} = 2491$ cps

$\varphi_1 = 330^o$

$\varphi_2 = 336^o$

These inputs lead to the following results

$v = 24.74$ knots

$\alpha = 352^o \ 20'$

It can be seen that the inaccuracies of measurement have produced some error in the calculated results. However, the error is surprisingly small, amounting to only about a quarter of a knot in speed and eight degrees in heading. Such accuracy certainly would not be sufficient for a fire control solution, but for the submarine 25 or 30 miles ahead of a convoy and maneuvering for position, it should be quite helpful. Furthermore, repeated recalculations of surface course and speed based on subsequent measurements of bearing and frequency could be expected to yield results which would be quite narrowly dispersed around the true values.

4. NO OWN-DOPPLER NULLIFICATION

If the C/P Array Sonar were to be operated without own-doppler nullification wherein pulses were transmitted at the same frequency independent of own-ship's course and speed, equation (5) above for the frequency of the sound received by the submarine in the ODN case would be modified to

$$f_{RS} \approx f_c \left[1 + \frac{v}{c} \cos \theta + \frac{v_s}{c} \cos \varphi\right] \tag{18}$$

where f_c now is the constant frequency of the transmitted signal.

Similarly, equations (9) and (10) would become

$$\cos \theta_1 \approx \frac{1}{v}\left[\left(\frac{f_{RS1}}{f_c} - 1\right) c - v_s \cos \varphi_1\right] \tag{19}$$

$$\cos \theta_2 \approx \frac{1}{v}\left[\left(\frac{f_{RS2}}{f_c} - 1\right) c - v_s \cos \varphi_2\right] \tag{20}$$

so that K_1 and K_2 are now modified to

$$K_1 = \left[\left(\frac{f_{RS1}}{f_c} - 1\right) c - v_s \cos \varphi_1\right] \tag{21}$$

$$K_2 = \left[\left(\frac{f_{RS2}}{f_c} - 1\right) c - v_s \cos \varphi_2\right] \tag{22}$$

The equations for θ_1, α, and v will be the same as shown previously in equations (15), (16), and (17) except that the values for K_1 and K_2 will be altered as shown above.

Thus, it can be seen that the submarine can calculate the course and speed of the surface vessel just as well when the surface vessel is transmitting at a constant frequency as when the frequency is being modified according to ship's speed and direction of transmission. In either case, it is necessary for the submarine to observe the direction and frequency of the incoming signal. For the ODN case, the frequency to which the C/P Array Sonar ship is holding the output of the receiving array must be known. In the non-ODN case, the frequency at which the ship is driving its transmitting array must be known.

5. DETERMINATION OF TRANSMITTED FREQUENCY

The solution for target course and speed in the ODN case requires a knowledge of the frequency to which the output of the receiving array is being held by the C/P Array Sonar. In the non-ODN case, the constant frequency at which the transmitting array is being driven must be known. It is possible that the submarine might have some knowledge through intelligence sources as to the operational frequencies of the sonar. However, if the option were built into the equipment to operate at any of several very slightly different frequencies, the submarine would have no way of knowing which frequency was in use at the moment. And if the differences were so slight as to produce solutions all of which were reasonable, the ambiguity could not be resolved by an inspection of the results.

It is possible, however, for the submarine to calculate the reference frequency from three measurements of bearing and frequency of the received signal. Details of the derivation are quite straight forward but rather cumbersome, so are not given here. They show that the frequency to which the ODN receiving array output is held will be given by

$$f_c = c\left[\frac{f_{RS1}\sin(\varphi_3-\varphi_2) + f_{RS2}\sin(\varphi_1-\varphi_3) + f_{RS3}\sin(\varphi_2-\varphi_1)}{k_1\sin(\varphi_3-\varphi_2) + k_2\sin(\varphi_1-\varphi_3) + k_3\sin(\varphi_2-\varphi_1)}\right] \qquad (23)$$

where $k_1 = c + v_s \cos \varphi_1$

$k_2 = c + v_s \cos \varphi_2$

$k_3 = c + v_s \cos \varphi_3$

f_{RS1} = frequency measured at sub-first observation

f_{RS2}= frequency measured at sub-second observation

f_{RS3}= frequency measured at sub-third observation

φ_1 = relative bearing sub to ship-first observation

φ_2 = relative bearing sub to ship-second observation

φ_3 = relative bearing sub to ship-third observation

c = velocity of sound in sea water

Figure 5-1 shows an extension of the geometry of Fig. 3-1 to include a third positioning of the ship and submarine. This shows

$$\theta_3 = 49^o \ 55'$$

$$\varphi_3 = 349^o \ 55'$$

Calculating from equation (5) the frequency received by the submarine in this case yields

$$f_{RS3} = 2494.61 \text{ cps}$$

Using this value along with the value for φ_3 and the other values previously given and solving for f_c using equation (23) gives the result

$$f_c = 2,499.07$$

Recalculating surface ship course and speed using this value for f_c instead of the value of 2500 which had just been assumed before yields

$$v = 23.75 \text{ knots}$$
$$\alpha = 358^o \ 22'$$

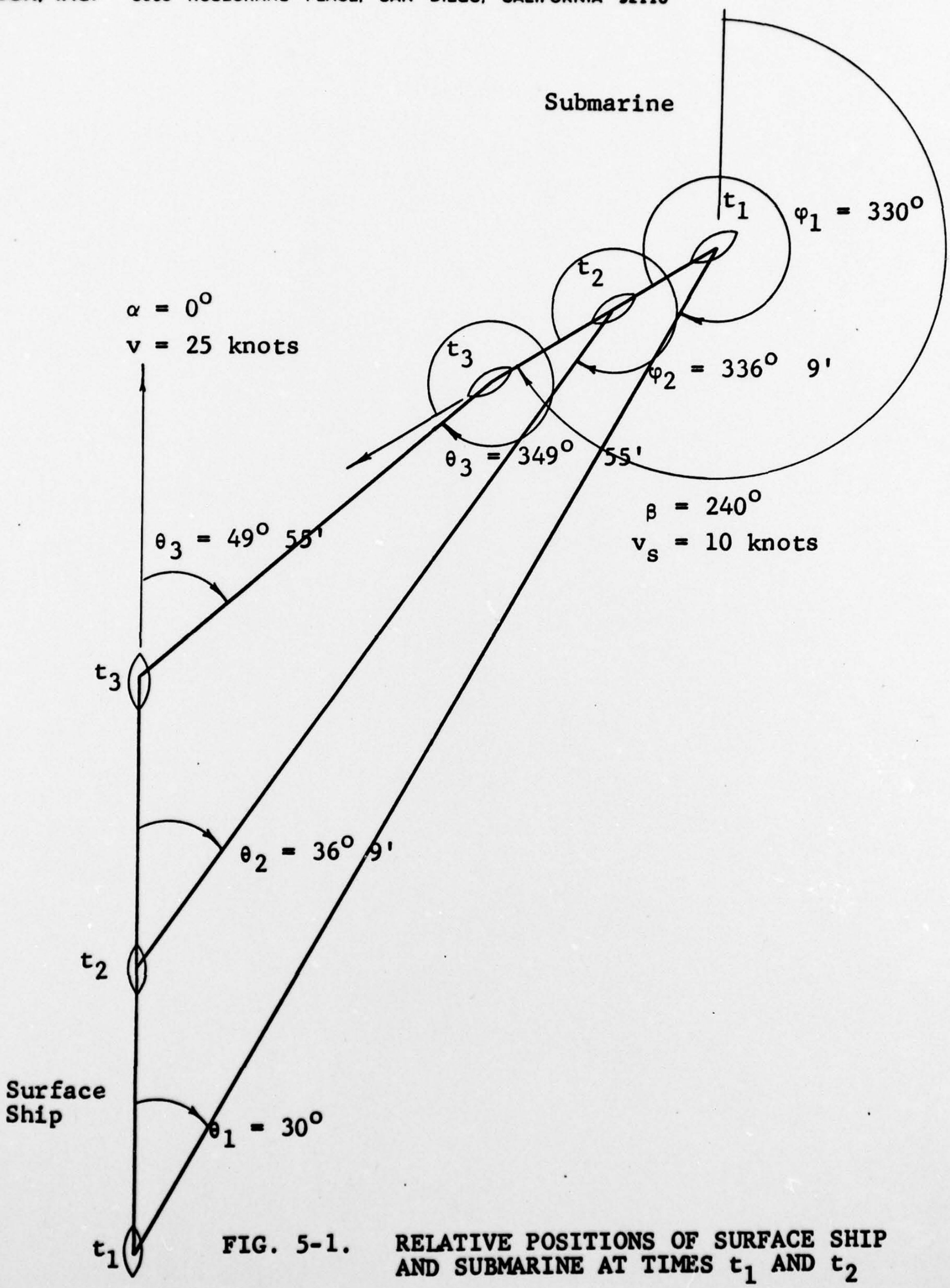

FIG. 5-1. RELATIVE POSITIONS OF SURFACE SHIP AND SUBMARINE AT TIMES t_1 AND t_2

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It is seen that this inaccuracy in the calculation of the reference frequency leads to a significant error in the values found for surface vessel course and speed. However, it can be expected that averaging numerous such calculations would reduce the error and yield solutions for course and speed which would be of great value in tactical maneuvering.

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6. DETERMINATION OF SURFACE VESSEL COURSE AND SPEED WITH HALF DOPPLER NULLIFICATION

Since the frequency into the water in the center-beam direction was shown in Eq. (1) to be

$$f_w = f_D \left[1 + \frac{v}{c} \cos \theta\right]$$

the frequency into the water will be constant at f_c if the transmitting array is driven at

$$f_D = \frac{f_c}{\left[1 + \frac{v}{c} \cos \theta\right]} \tag{24}$$

Thus, the submarine would always hear the same frequency signal, independent of surface vessel course and speed, when the sonar beam was aimed at the submarine. In modifying Eq. (5) now for the frequency of sound received by the submarine, the equation degenerates to

$$f_{RS} = f_c \left[1 + \frac{v_s}{c} \cos \varphi\right] \tag{25}$$

Since this equation is not a function of θ, equations for $\cos \theta_1$ and $\cos \theta_2$, comparable to (9) and (10), can no longer be written. This necessitates approaching the problem in a different manner.

Rewriting Eq. (25) in terms of f_c

$$f_c = \frac{f_{RS}}{1 + \frac{v_s}{c} \cos \varphi} \tag{26}$$

and substituting this value for f_c into Eq. (24) yields

$$f_D = \frac{f_{RS}}{\left[1 + \frac{v_s}{c}\cos\varphi\right]\left[1 + \frac{v}{c}\cos\theta\right]}$$

or

$$f_{RS} \approx f_D\left[1 + \frac{v}{c}\cos\theta + \frac{v_s}{c}\cos\varphi\right] \qquad (27)$$

The similarity between this equation and Eq. (5) is quite striking. Just as Eqs. (9) and (10) were written by solving Eq. (5) for $\cos\theta_1$ so can the following two equations be written by solving Eq. (27) for $\cos\theta$

$$\cos\theta_1 = \frac{1}{v}\left[\left(\frac{f_{RS1}}{f_{D1}} - 1\right)c - v_s\cos\varphi_1\right] \qquad (28)$$

$$\cos\theta_2 = \frac{1}{v}\left[\left(\frac{f_{RS2}}{f_{D2}} - 1\right)c - v_s\cos\varphi_2\right] \qquad (29)$$

Once again, these equations are quite similar to (9) and (10); and at first glance, it would appear that if surface ship course and speed could be found from (9) and (10), they could be found from (28) and (29). However, there is one important difference. Equations (9) and (10) involve f_c, the frequency to which the surface ship is holding the output of its receiving array. It was assumed that this frequency was known to the submarine and was constant. It was later shown that this frequency could be calculated by the submarine from a third observation of bearing to the surface ship and the frequency of the received signal.

But the assumption was still made that the frequency f_c was constant. In the present case, f_D is not only not known but will not be constant from observation to observation if the geometry is changing as a function of time. Thus, whereas there were two equations and three unknowns (θ_1, θ_2, and v) in (9) and (10), there are two equations and five unknowns (θ_1, θ_2, v, f_{D1}, and f_{D2}) in (28) and (29). The addition of Eq. (8), ($\theta_2 = \theta_1 + \Delta\varphi$) rendered the previous case solvable whereas it leaves three equations and five unknowns in the present case. Taking an observation at a third point would add two equations

$$\cos \theta_3 = \frac{1}{v}\left[\left(\frac{f_{RS3}}{f_{D3}} - 1\right) c - v_s \cos \varphi_3\right]$$

$$\theta_3 - \theta_2 = \varphi_3 - \varphi_2 = \Delta\varphi_{32}$$

but, it would also add two unknowns (θ_3 and f_{D3}), so it would not bring solution any nearer. It appears that a solution for surface ship course and speed is impossible working from center-beam observations in the case where the surface ship modifies its transducer driving frequency to put a constant center-beam frequency into the water. The possibility of solving for surface ship course and speed by using side-lobe information will be investigated in a subsequent paper.

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7. CONCLUSIONS

1. If the surface vessel modifies the frequency at which its transmitting array is driven to give constant center-beam frequency into the water independent of ship speed and direction of transmission (Half Doppler Nullification--HDN), the submarine will be unable to calculate surface vessel course and speed from center-beam bearing and frequency readings. A subsequent paper will investigate to determine if transmit beam side-lobe information will enable the submarine to determine surface ship course and speed.

2. If the surface vessel modifies the frequency at which its transmitting array is driven to give a constant frequency at the output of its receiving array (Full Doppler Nullification--FDN), the submarine will be able to calculate the course and speed of the surface vessel from three or more observations of center-beam frequency and bearing to the vessel.

3. If the surface vessel drives its transmitting array at constant frequency (No Doppler Nullification--NDN), the submarine will be able to calculate the course and speed of the surface vessel from three or more observations of center-beam frequency and bearing to the vessel.

4. The accuracy requirements in measuring bearing and frequency of signals received at the submarine in the Full Doppler and No Doppler Nullification cases are too high to yield a solution good enough for fire control purposes. However, it should be possible to determine surface vessel course and speed sufficiently well to be of considerable value in tactical maneuvering.